SYLLABUS FOR Ph.D ENTRANCE TEST
MATHEMATICS

UNIT- 1 ALGEBRA

Groups, Lagrange's Theorem, Homomorphism and isomorphism, Normal subgroups and factor groups, The fundamental theorem of homomorphism, Two laws of isomorphism, Permutation groups and Cayley's theorem, Sylow's theorems.

Rings, Integral domains and Fields, Homomorphisms, Ideals and Quotient Rings, Prime and Maximal ideals, Euclidean and principal ideal rings, Polynomials, Zeros of a polynomial, Factorization, Irreducibility criterion, Adjunction of roots, Algebraic and transcendental extensions, Finite fields, Separable and inseparable extensions, Perfect and imperfect fields, Theorem on the primitive element.

Vector Spaces, Subspaces, Linear combinations and systems of linear equations, Linear dependence and linear independence, Bases and dimension, Maximal linearly independent subsets.

Linear transformations, Null spaces and ranges, The matrix representation of a linear transformation, Composition of linear transformations and matrix multiplication, Invertibility and isomorphisms, The change of coordinate matrix, The dual space, Eigenvalues and eigenvectors, Diagonalizability, Invariant subspaces and the Cayley-Hamilton theorem, Inner products and norms, The Gram-Schmidt orthogonalization process and orthogonal complements, The adjoint of a linear operator, Normal and self-adjoint operators, Unitary and orthogonal operators and their matrices, Orthogonal projections and the spectral theorem, Bilinear and quadratic forms, The diagonal form, The triangular form, The Jordan canonical form; The minimal polynomial, The rational canonical form.

UNIT-2 REAL ANALYSIS

Numerical sequences, Convergent sequences, Cauchy sequences,Upper and lower limits. Series of real numbers series of non-negative terms, the number 'e', tests of convergence, Multiplications of series, Re-arrangements, Double series, infinite products.

Finite, countable and uncountable sets, The topology of the real line. Continuity, Uniform continuity, Properties of continuous functions, Discontinuities, Monotonic functions,

Differentiability, Mean value theorems, L' Hospital rule, Taylor's theorem, Maxima and minima, Functions of bounded variation, The Riemann-Stieltje's integral, Criterion for integrability, Properties of the integral, Classes of integrable functions, The integral as the limit of a sum, First and second mean value theorems, Integration and differentiation.

Sequences and series of functions, Uniform convergence, Uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation, Power series, The exponential and logarithmic functions, The trigonometric functions.

Improper integrals and their convergence, Functions of several variables, Partial derivatives, Continuity and differentiability, The chain rule, Jacobians, The Implicit function theorem, Taylor's theorem, The maxima and minima, Lagrange's multipliers.

UNIT-3 COMPLEX ANALYSIS

Analytic functions, Cauchy-Riemann equations, Harmonic functions, Polynomials and Rational functions, Elementary theory of power series, sequences, series, uniform convergence of power series, Abel's limit theorem, The elementary functions, Topology of the complex plane, Linear fractional transformations, Cross-ratio, Symmetry, Elementary conformal mappings, Line integrals, Rectifiable arcs, Local properties of analytic functions, Cauchy's theorem for a rectangle, Cauchy's theorem in a Circular disk, Cauchy's integral formula.

The residue theorem, Argument principle, Evaluation of definite integrals, Harmonic functions - Definition and basic properties, Mean value property, Poisson's formula, Schwarz's theorem, Reflection principle, Power series expansions , The Weierstrass theorem, The Taylor series, The Laurent series.

Partial fractions, Mittag - Leffer's theorem, Infinite products, Canonical products, The Gamma and Beta functions, Sterling's formula, Entire functions, Jensen's formula, Hadamard's theorem.

UNIT-4 ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS

Linear Second Order Equations, Initial value problem, Existence and uniqueness by Picard's theorem, Wronskian, Separation and comparison theorems, Poincare phase plane analysis, Method of variation of parameters.

Power series solutions, Solution near ordinary and regular singular point, Convergence of the formal power series, applications to Legendre, Bessel, Hermite, Laguerre and hypergeometric differential equations with their properties.

Partial differential equations, Cauchy problems and characteristics, Classification of second order PDE's, Reduction to canonical forms, Equations of mathematical physics and their solutions.

Boundary value problems, Transforming Boundary value problem of PDE and ODE, Sturm - Liouvile system, Eigen values and eigen functions, Simple properties, Fourier expansion in eigen functions, Parseval's identity, Green's function method.

UNIT-5 TOPOLOGY

Topological spaces, Basis for a topology, The order topology, The product topology on $X \times Y$, The subspace topology, Closed sets and limit points, Continuous functions, The product topology, The metric topology, The quotient topology, Connected spaces, Connected sets on the real line, Path connectedness, Compact spaces, Compact sets on the line, Limit point compactness, Local compactness.

The countability axioms, The separation axioms, Urysohn's lemma, Tietze's extension theorem, Urysohn's metrization theorem, Partitions of unity, Tychonoff's theorem on the product of compact spaces. Local finiteness, Paracompactness, Normality of a paracompact space.

The Fundamental group and the fundamental group of a circle, The fundamental group of the punctured plane, Essential and inessential maps, The fundamental theorem of algebra.

UNIT-6 ELEMENTS OF FUNCTIONAL ANALYSIS

Metric completion, Banach's contraction mapping theorem and applications, Baire' category theorem, Ascoli - Arzela theorem.

Linear spaces and linear operators, Norm of a bounded operator, The Hahn - Banach extension theorem, Stone - Weirstrass theorem.

Open mapping and closed graph theorems, Principle of Uniform Boundedness.

Hilbert spaces- The orthogonal projection, Nearly orthogonal elements, Riesz's lemma, Riesz's representation theorem.

UNIT-7 DIFFERENTIAL GEOMETRY

Plane curves and space curves, Frenet-Serret formulae, Simple closed curves, The isoperimetric inequality, The four vertex theorem, Smooth surfaces, Tangents, Normals and orientability, Quadric surfaces, The lengths of curves on surfaces, Isometries of surfaces, Conformal mappings of surfaces, Surface area, Equiareal maps and a theorem of Archimedes.

Curvature of surfaces - The second fundamental form, The curvature of curves on a surface, Normal and principal curvatures.

The Gaussian and the mean curvatures, The Pseudo sphere, Flat surfaces, Surfaces of constant mean curvature, Gaussian curvature of compact surfaces, The Gauss' map.

UNIT-8 MEASURE AND INTEGRATION

Outer measure, Measurable sets and Lebesgue measure, A nonmeasurable set, Measurable functions.

The Lebesgue Integral of a bounded function over as set of finite measure, The integral of a non-negative function, The general Lebesgue integral.

Differentiation of monotonic functions, Functions of bounded variation, Differentiation of an integral, Absolute continuity.

Measure spaces, Measurable functions, Integration, Signed measures, The Radon - Nikodym theorem, Measure and outer measure, Outer measure and measurability, The extension theorem, Product measures.

UNIT-9 NUMBER THEORY

Congruences, Residue classes, Theorems of Fermat, Euler and Wilson, Linear congruences, Elementary arithmetical functions, Primitive roots, Quadratic residues and the law of quadratic reciprocity, Prime numbers, The Fundamental theorem of arithmetic, The series of reciprocals of primes, The Euclidean algorithm, Fermat and Mersenne numbers, Farey series, Farey dissection of the continuum, Irrationality of m^{th} root of N, e and π.

The Mobius function, The Euler' function and Sigma function, The Dirichlet product of arithmetical functions, Multiplicative functions, Euler summation formula, Some elementary asymptotic formulas, The average orders of $d(n), \sigma(n), \phi(n), \mu(n)$. An application to the distribution of lattice points visible from the origin.

Finite continued fractions, Convergent of a continued fraction, Continued fractions with positive quotients. Simple continued fractions, The representation of an irreducible rational fraction by a simple continued fraction, The continued fraction algorithm and Euclid's algorithm. The difference between the fraction and its convergents, Infinite simple continued fractions, The representation of an irrational number by an infinite continued fraction, Equivalent numbers and periodic continued fractions, Some special quadratic surds.

UNIT-10 GRAPH THEORY

Graphs, Vertices of graphs, Walks and connectedness, Degrees, Operations on graphs, Blocks , Cutpoints, Bridges, Cutpoint graphs, Sub-graphs, Paths and cycles, Connected graphs, Connected components, Adjacency and incidence matrices.

Properties of trees, Center, Connectivity, Connectivity and line connectivity, Menger's theorem, Partitions, Coverings and independence number.

Euler tours, Euler graphs, Hamiltonian paths, Hamiltonian graphs ,Closure of a graph, Planar graphs, Euler's formula, Vertex colouring, Chromatic number, Chromatic polynomial , R - Critical graphs.