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Estd. 1916

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**NOTIFICATION**

01-6

Sub: Introduction of two new soft core courses namely "Modular Functions" and "Hypergeometric functions and q-series" for the III semester and IV semester M.Sc. Mathematics from the Academic Year 2016-17.

Ref: 1. Decision of the Faculty of Science & Technology Meeting held on 16.02.2016.

2. Decision of the Academic Council meeting held on 29-03-2016.

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The Board of Studies in Mathematics (PG) which met on 9-11-2015 has resolved to Introduce two new soft core courses namely "Modular Functions" and "Hypergeometric functions and q-series" for the III semester and IV semester M.Sc. Mathematics. (PG) as follows from the academic year 2016-17.

**III Semester M. Sc Mathematics Soft Core Syllabus**

Course Title: **Modular Functions**

Teaching Hours: 5/week

**Unit 1. Elliptic Functions**

Introduction, Doubly periodic functions, Fundamental pairs of periods, Elliptic functions, Construction of elliptic functions, The Weierstrass  $p$  function, The Laurent expansion of  $p$  near the origin, Differential equation satisfied by  $p$ , The Eisenstein series and the invariants  $g_2$  and  $g_3$ , The numbers  $e_1, e_2, e_3$  The discriminant  $\Delta$ , Klein's modular function  $J(\tau)$ , Invariance of  $J$  under unimodular transformations, The Fourier expansions of  $g_2(\tau)$  and  $g_3(\tau)$ , The Fourier expansions of  $\Delta(\tau)$  and  $J(\tau)$ .

**Unit 2. The Modular group and modular functions**

Mobius transformations, The modular group  $\Gamma$ , Fundamental regions, Modular functions, Special values of  $J$ , Modular functions as rational functions of  $J$ , Mapping properties of  $J$ , Application to the inversion problem for Eisenstein series, Application to Picard's theorem.

**Unit 3. The Dedekind eta function**

Introduction, Siegel's proof of Theorem 3.1, Infinite product representation for  $\Delta(\tau)$ , The general functional equation for  $\eta(\tau)$  transformation formula, Deduction of Dedekind's functional equation from Iseki's Formula, Properties of Dedekind sums, The reciprocity law for Dedekind sums, Congruence properties of Dedekind sums, The Eisenstein series  $G_2(\tau)$ .

#### **Unit 4. Congruences for the coefficients of the modular function $j$**

Introduction, The subgroup  $\Gamma_0(q)$ , Fundamental region of  $\Gamma_0(p)$ , Functions automorphic under the subgroup  $\Gamma_0(p)$ , Construction of functions belonging to  $\Gamma_0(p)$ , The behavior of  $j_p$  under the generators of  $\Gamma$ , The function  $\varphi(\tau) = \Delta(q\tau)/\Delta(\tau)$ , The univalent function  $\phi(\tau)$ , Invariance of  $\phi(\tau)$  under transformations of  $\Gamma_0(q)$ , The function  $j_p$  expressed as a polynomial in  $\phi$ .

#### **Books for Reference:**

1. Tom M. Apostol, *Modular Functions and Dirichlet Series in Number Theory*, Springer-Verlag, 1976..
2. Gunning, R. C. *Lectures on Modular Forms*, Annals of Mathematics Studies, No. 48. Princeton Univ. Press, Princeton, New Jersey, 1962. MR 24 # A2664.

### **IV Semester M. Sc Mathematics Soft Core Syllabus**

**Course Title: Hypergeometric Functions and  $q$ -Series.**      **Teaching Hours: 5/week**

#### **Unit 1. The Gauss Function**

Historical introduction, The Gauss series and its convergence, The Gauss equation, The connection with Riemann's equation, Kummer's twenty-four solutions, Contiguous functions and recurrence relations, Special cases of the Gauss function, Some integral representations, The Gauss summation theorem, Another special summation theorem, Analytic continuation formulae.

#### **Unit 2. Basic Hypergeometric Functions**

Convergence of Heine series-Some simple results, Jackson's Theorem, Basic analogue of Saalschutz's Theorem, Application of Bailey's transformation to basic series, Some numerical evaluation of infinite products, Basic bilateral series, Ramanujan's  ${}_1\Psi_1$  summation formula.

#### **Unit 3. Theta Functions**

Ramanujan's general theta-function and its particular cases, Theta-function identities of Ramanujan found in his Chapter 16 of his second notebook, Quintuple product identity and its applications.

#### **Unit 4. $q$ -Continued fractions**

Ramanujan's cubic continued fractions, Rogers-Ramanujan continued fractions and related theta-function identities.

#### **Books for Reference:**

1. L. J. Slater, *Generalized Hypergeometric Functions*, Cambridge University Press, London, 1966.
2. H. Exton,  *$q$ -Hypergeometric Functions and Applications*, Ellis Horwood Series in Mathematics and its Application, Chichester, 1983.