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D-Integrity and E-Integrity Numbers in Graphs

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Abstract: Inspired by the definition of integrity and the alternative formulations for integrity, we investigate the $D-$Integrity and $E-$Integrity numbers of a graph in the present study. The D-Integrity number of a graph G is denoted by $DI_k(G)$ defined as: $DI_k(G) = \sum_{k=1}^{p} D_k(G)$, and the E-Integrity number of a graph G, is denoted by $EI_l(G)$ defined as: $EI_l(G) = \sum_{l=0}^{p} E_l(G)$. In this paper, we establish the general formulas for the $D-$Integrity and $E-$Integrity numbers of some classes of graphs. Also, some properties of $D-$Integrity and $E-$Integrity numbers are established.

Keywords: Integrity, D-Integrity number, E-Integrity number

1 Introduction

Throughout this paper, we consider simple and undirected graphs. Let $G = (V,E)$ be such a graph. The number of vertices of G is denoted by p and the number of edges is denoted by q, so $|V(G)| = p$ and $|E(G)| = q$. The degree of a vertex v, denoted by $deg(v)$, is the number of vertices adjacent to v. The contraction of a vertex v in G (denoted by G/v) is the graph obtained by deleting v and putting a clique on the open neighbourhood of v, (note that this operation does not create multiple edges, and if two neighbours of v are already adjacent, they remain simply adjacent) [1]. A spider graph G_s is a tree which is constructed by subdividing each edge once in $K_{1,p-1}, p \geq 3$ [2]. If every pair of vertices of a graph G are adjacent, G is called a complete graph, and it is denoted by K_p with p vertices.

A graph G is called a bipartite graph if the vertex set V can be partitioned into two subsets V_1 and V_2 such that every edge of G joins a vertex of V_1 with a vertex of V_2. Furthermore, if every vertex of V_1 joins every vertex of V_2, G is a complete bipartite graph. The complete bipartite graph with two sets of vertices such that $|V_1| = n$, and $|V_2| = m$ is denoted by $K_{n,m}$. The graph $K_{1,p-1}$ is a star, or a star is a tree with at most one non-pendant vertex. If u and v are not adjacent vertices in G, the addition of edge $e = (u,v)$ yields the graph $G+e$. For the vertex set V and edge set $E \cup \{e\}$. For the terminology not defined here, we refer the reader to [3].

A network can be modelled by a graph whose vertices represent the nodes and edges represent the lines of communication. Its efficiency reduces when some vertices or edges are destroyed anyway. Various graph parameters have been used to describe the vulnerability of communication networks (graph), like connectivity, tenacity, and integrity. The concept of integrity of a graph G was introduced in [4] as a useful measure of the vulnerability of G. The authors in [4] compared integrity, connectivity, toughness and binding number for several classes of graphs. Their results suggested that integrity is appropriate for measuring vulnerability, and so it can distinguish between graphs that should have different measures of vulnerability. The integrity of a graph G is defined as

$$I(G) = \min\{|S| + m(G-S) : S \subseteq V(G)\},$$

where $m(G-S)$ denotes the order of the largest component of $G-S$. An I-set of G is any subset S of $V(G)$ for which

$$I(G) = |S| + m(G-S).$$

For more about integrity, see [5,6]. The authors in [7,8,9, 10,11,12] introduced the new concepts of integrity parameter. In (1990), Goddard and Swart [13] introduced two concepts that are useful computationally as follows:

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